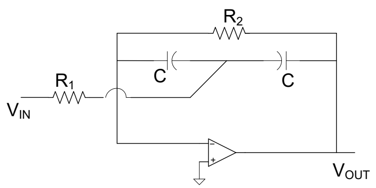
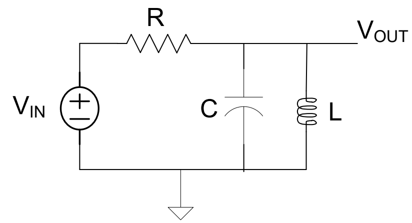
EE 508
Lecture 18

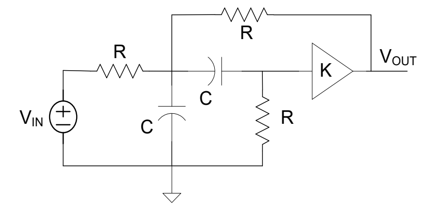
Comparison of Filter Structures

Sensitivity Functions

How does the performance of these bandpass filters compare?

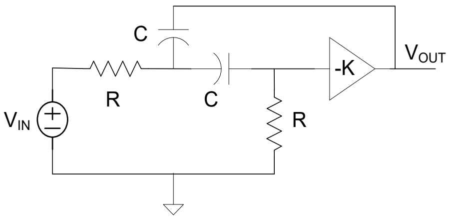


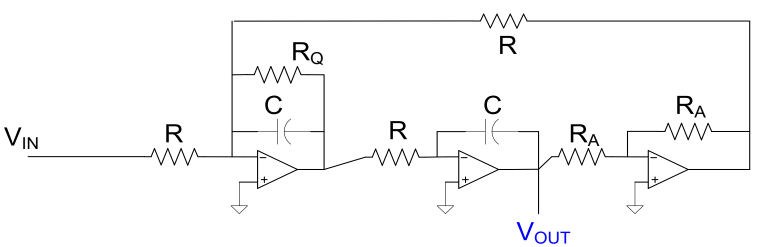




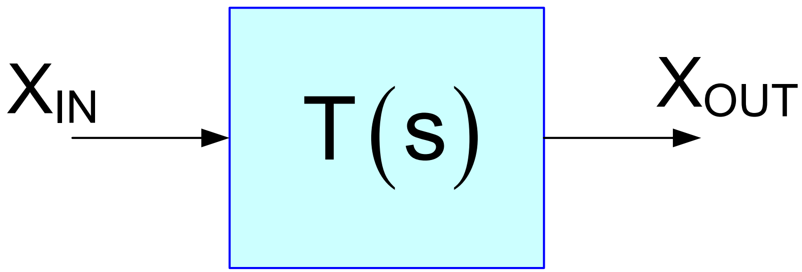
Review from last time

- Component Spread
- Number of Op Amps
- Is the performance strongly dependent upon how DOF are used?
- Ease of tunability/calibration (but practical structures often are not calibrated)
- Total capacitance or total resistance
- Power Dissipation
- Sensitivity
- Effects of Op Amps





Consider 2nd Order Lowpass Biquads



$$\left|T(s)\right| = H\frac{\omega_0^2}{s^2 + s\left(\frac{\omega_0}{Q}\right) + \omega_0^2}$$

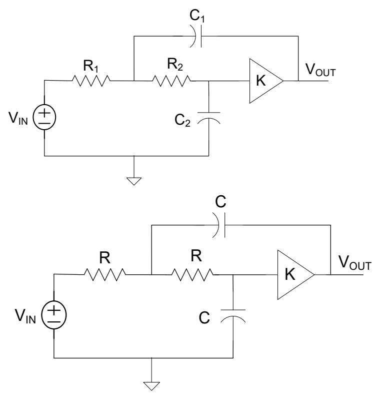
$$BW = \omega_B - \omega_A \neq \frac{\omega_0}{Q}$$

$$\omega_{PEAK} \neq \omega_0$$

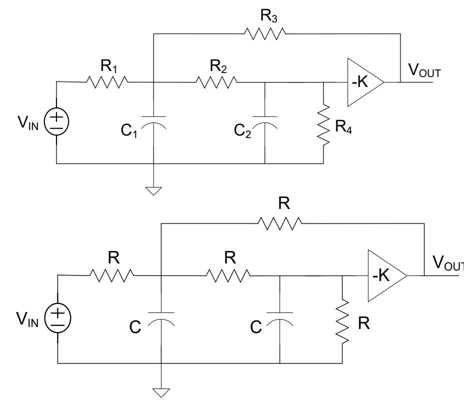
Consider 2nd Order Lowpass Biquads

$$\left|T(s)\right| = H\frac{\omega_0^2}{s^2 + s\left(\frac{\omega_0}{Q}\right) + \omega_0^2}$$

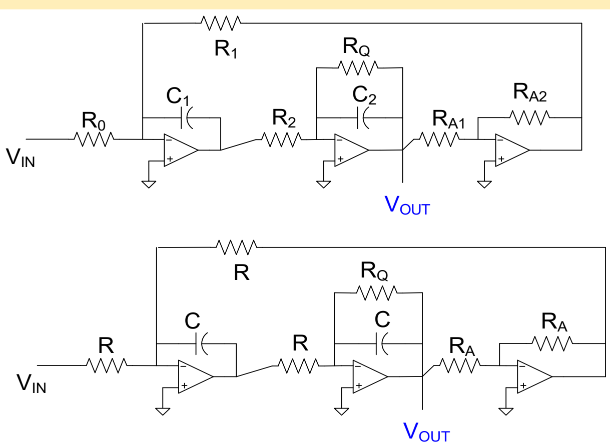
Four basic structures that ideally implement the same transfer function



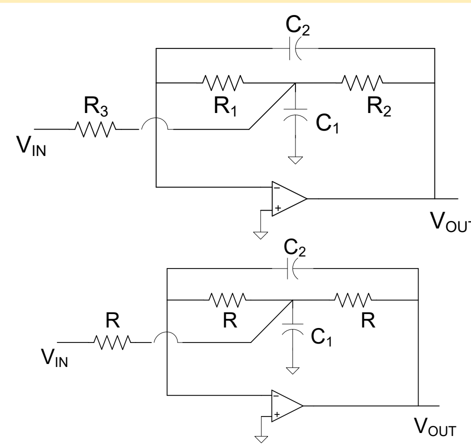
Sallen and Key +KRC



Sallen and Key -KRC

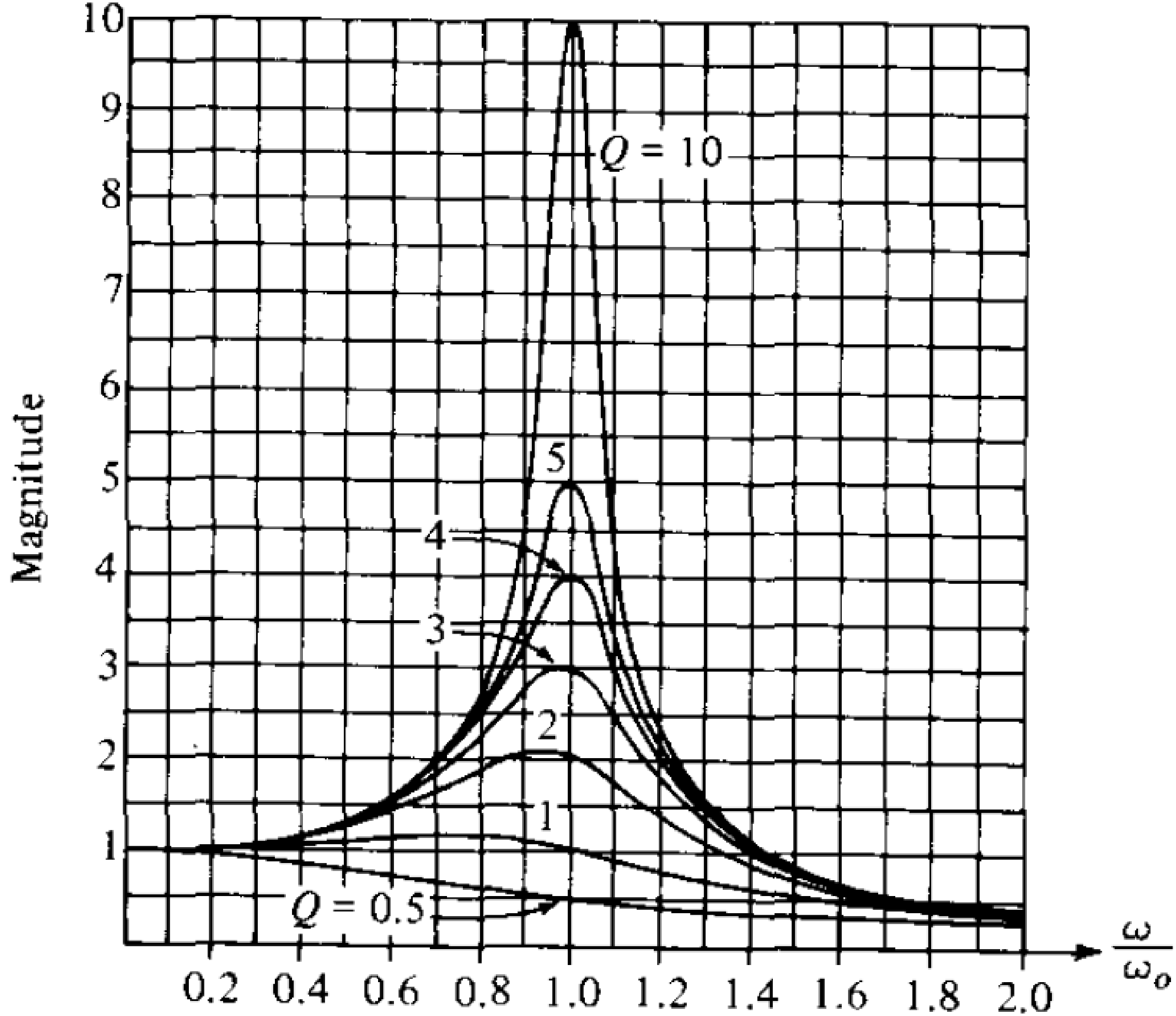


Two Integrator Loop

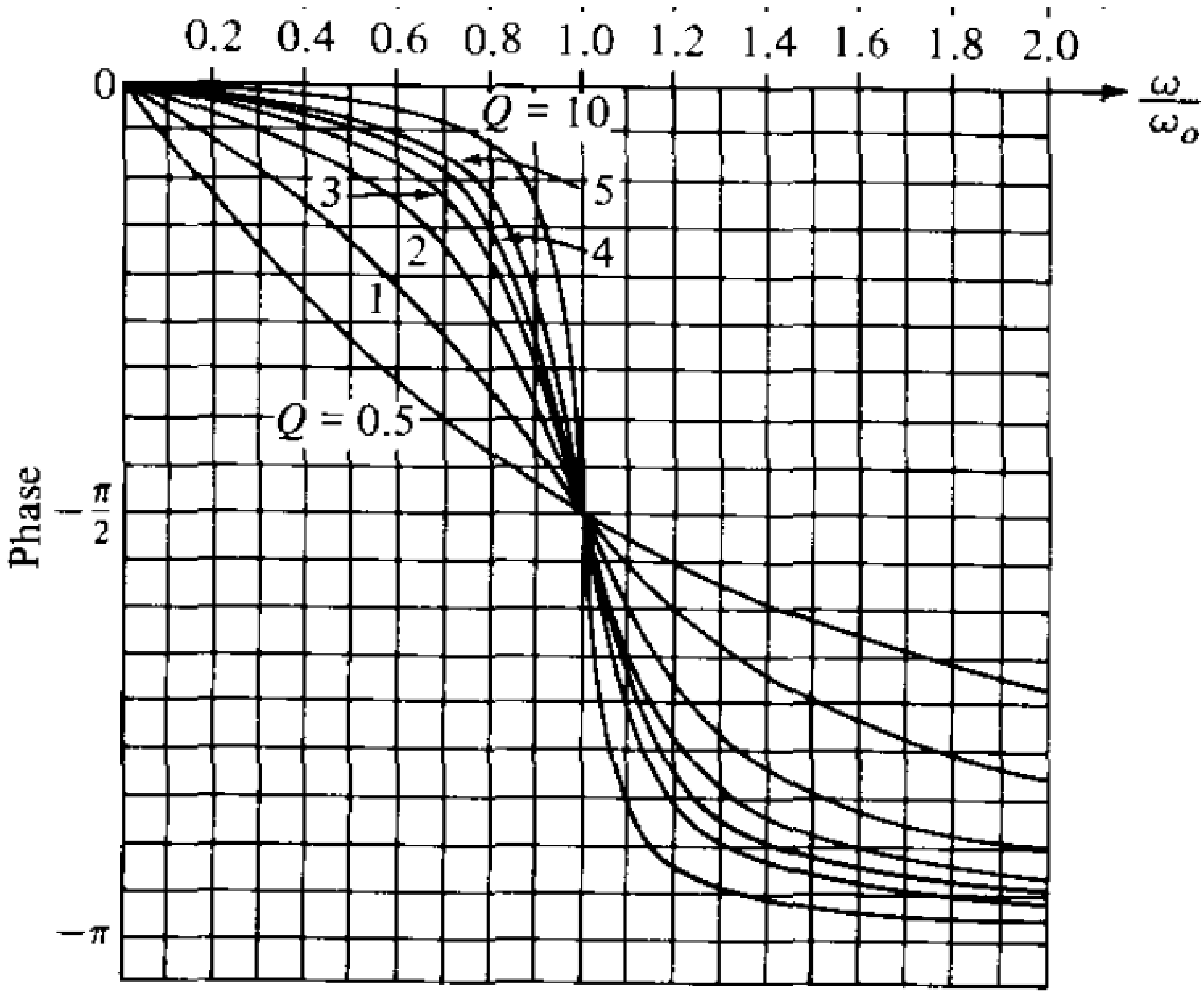


Bridged-T Feedback

Consider 2nd Order Lowpass Biquads



Consider 2nd Order Lowpass Biquads



Performance Comparison of Selected Second-Order Lowpass Filters

$$\left|T(s)\right| = H\frac{\omega_0^2}{s^2 + s\left(\frac{\omega_0}{Q}\right) + \omega_0^2}$$

$$BW = \omega_B - \omega_A \neq \frac{\omega_0}{Q}$$

$$\omega_{PEAK} \neq \omega_0$$

- Component Spread
- Number of Op Amps
- Is the performance strongly dependent upon how DOF are used?
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- Total capacitance or total resistance
- Power Dissipation
- Sensitivity
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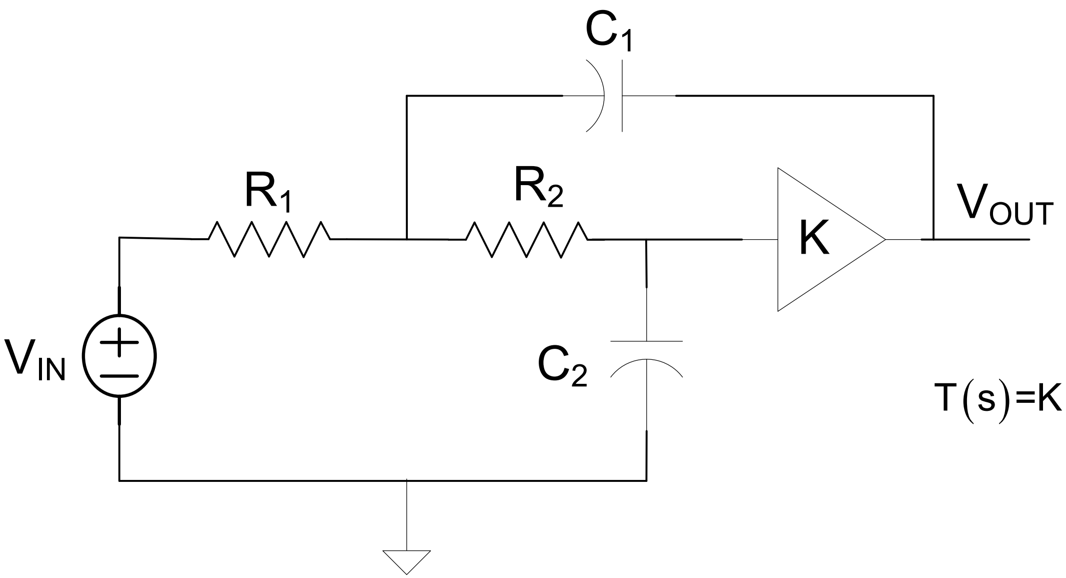
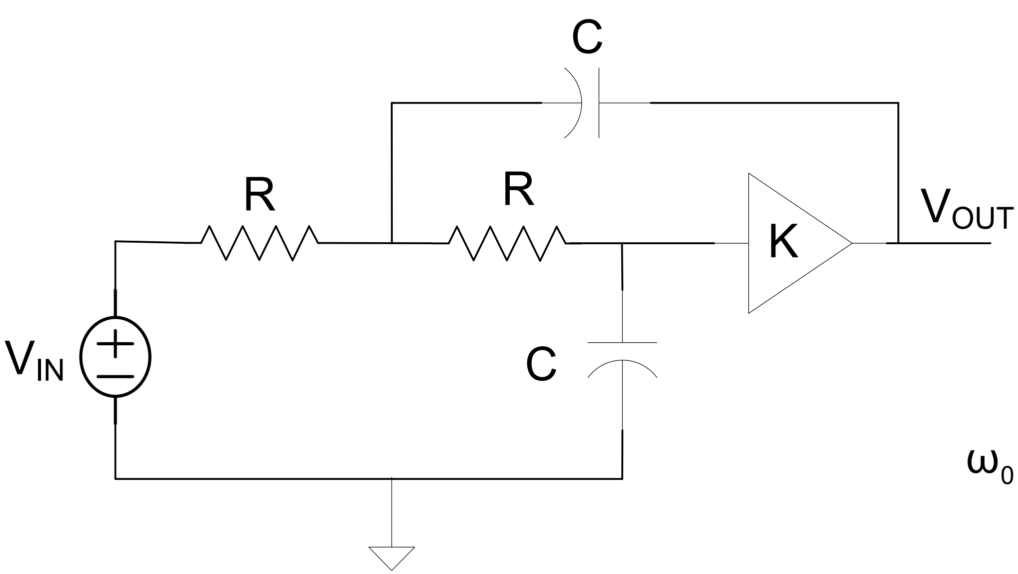
Example: 2nd Order +KRC Lowpass



$$T(s)=K\frac{\frac{1}{R_1R_2C_1C_2}}{s^2+s\left[\frac{1}{R_1C_1}+\frac{1}{R_2C_1}+\frac{1-K}{R_2C_2}\right]+\frac{1}{R_1R_2C_1C_2}}$$



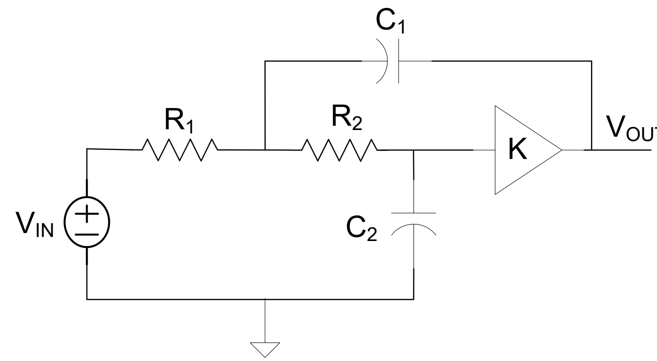
Equal R, Equal C

$$T(s)=K\frac{\frac{1}{R^2C^2}}{s^2+s\left[\frac{(3-K)}{RC}\right]+\frac{1}{R^2C^2}}$$

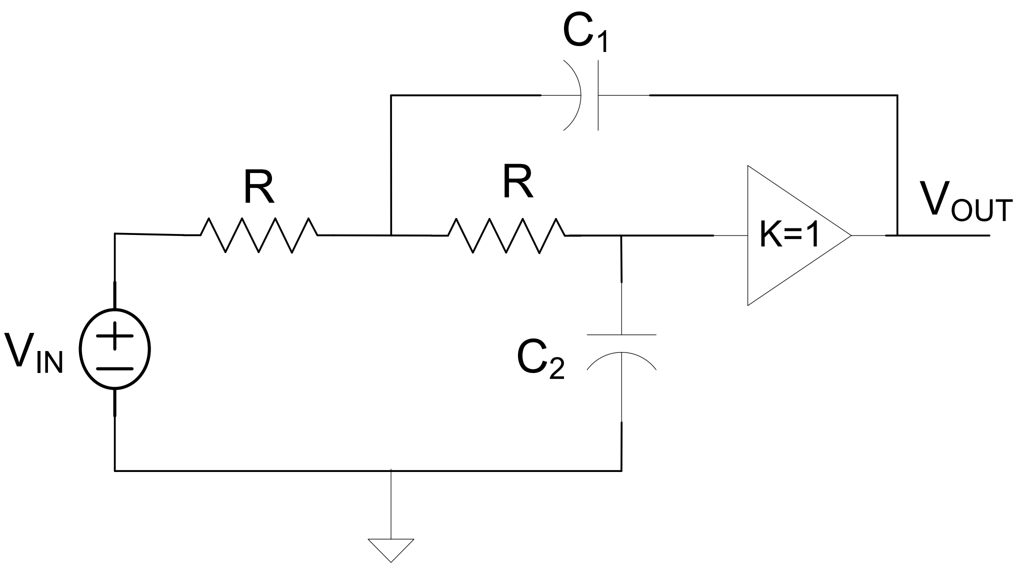
$$\omega_0=\frac{1}{RC}$$

$$Q=\frac{1}{3-K}$$

Example: 2nd Order +KRC Lowpass



Equal R, K=1



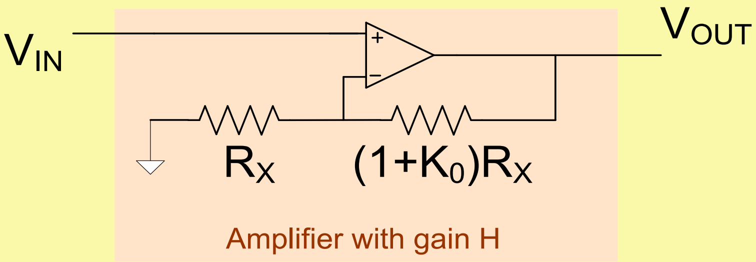
$$T(s)=K\frac{\frac{1}{R^2C_1C_2}}{s^2+s\left[\frac{2}{RC_1}\right]+\frac{1}{R^2C_1C_2}}$$

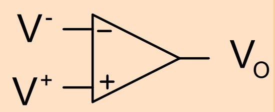
$$\omega_0=\frac{1}{R\sqrt{C_1C_2}}$$

$$Q=\frac{1}{2}\sqrt{\frac{C_1}{C_2}}$$

Example: 2nd Order +KRC Lowpass

Op Amp Model





$$A(s) = \frac{V_0}{V^+ - V^-} = \frac{GB}{s}$$

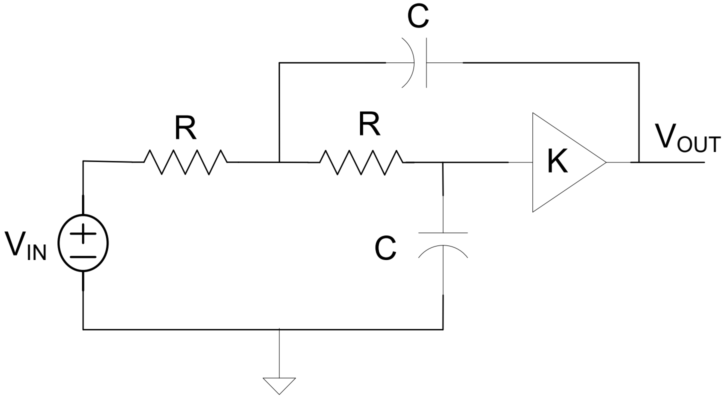
$$K(s) = \frac{K_0}{1 + \frac{K_0}{GB} s}$$

$$T(s) = H \frac{\omega_0^2}{s^2 + s\left(\frac{\omega_0}{Q}\right) + \omega_0^2}$$

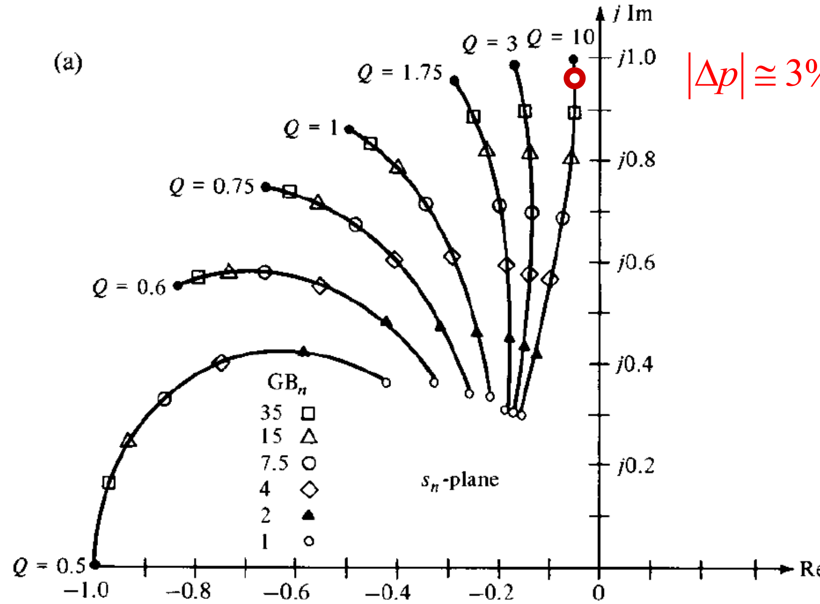
Define $$GB_n = \frac{GB}{\omega_0}$$

Example: 2nd Order +KRC Lowpass

Root Locus Plots



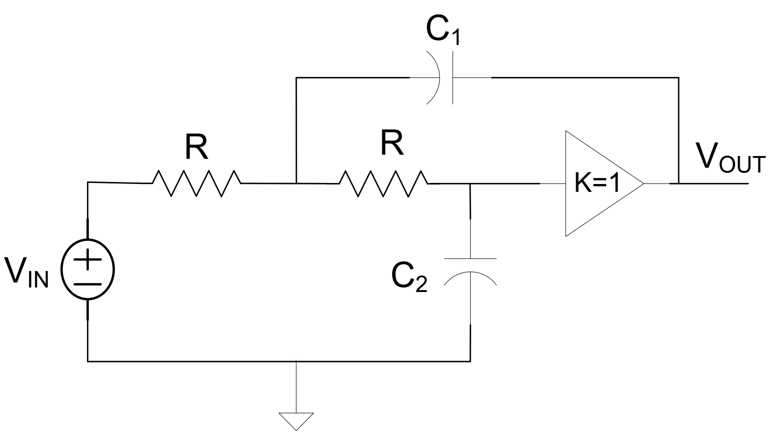
Equal R, Equal C



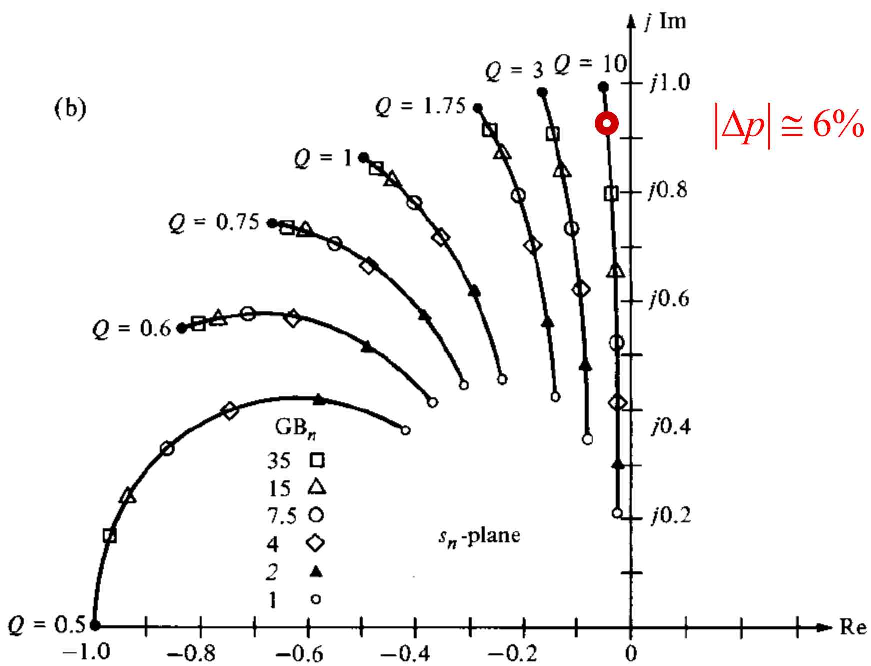
$|\Delta p| \cong 3\%$

consider

$\odot \longleftrightarrow GB_n = \frac{GB}{\omega_0} = 100$

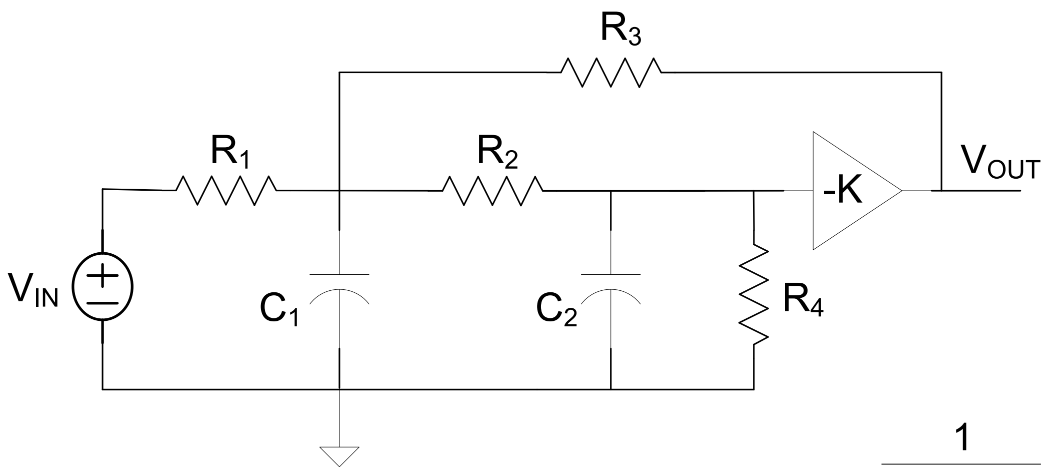


Equal R, K=1

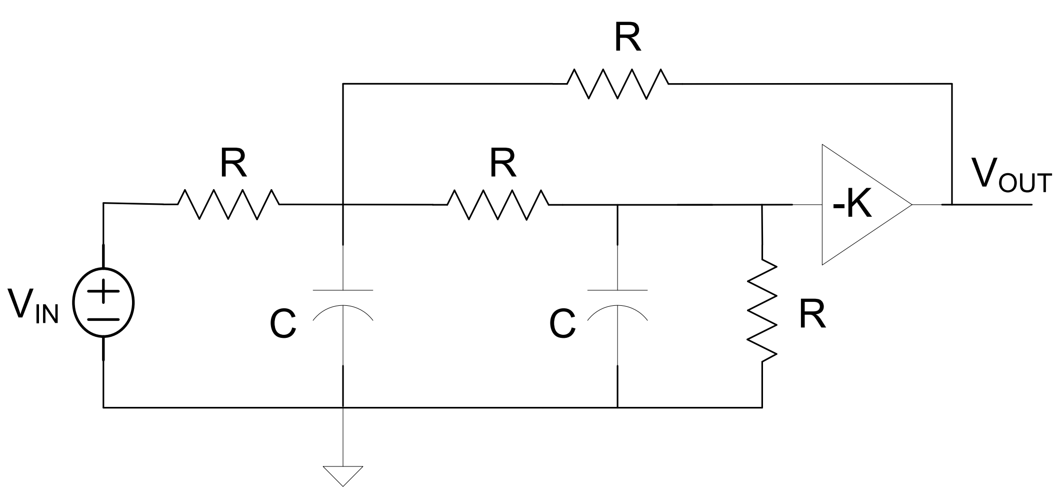


$|\Delta p| \cong 6\%$

Example: 2^{nd} Order -KRC Lowpass



$$T(s) = -K\frac{\frac{1}{R_1R_2C_1C_2}}{s^2+s\left[\frac{1}{R_1C_1}\left(1+\frac{R_1}{R_3}\right)+\frac{1}{R_4C_2}+\frac{1}{R_2C_2}\left(1+\frac{C_2}{C_1}\right)\right]+\left[\frac{1+(R_1/R_3)(1+K)+(R_1/R_4)\left(1+(R_2/R_3)+(R_2/R_1)\right)}{R_1R_2C_1C_2}\right]}$$

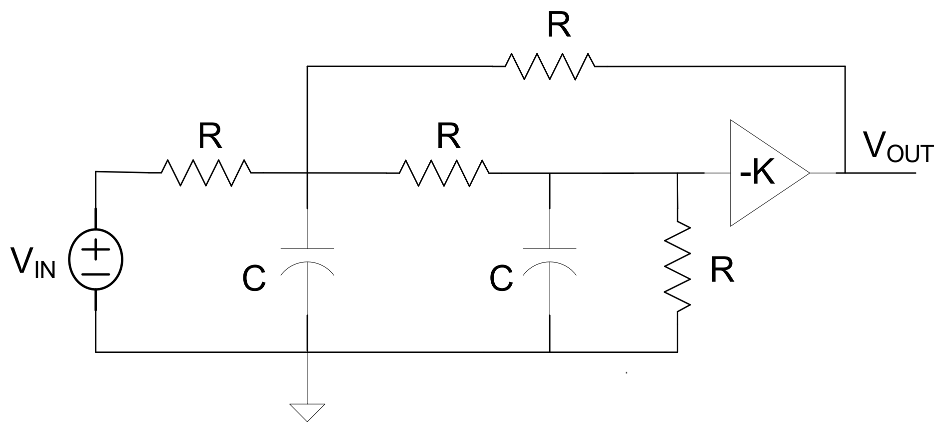


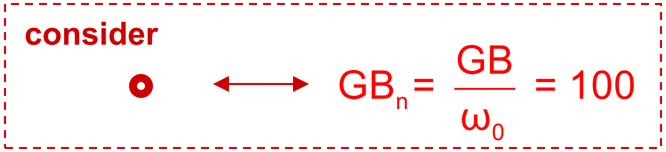
Equal R, Equal C

$$T(s) = -K\frac{\frac{1}{R^2C^2}}{s^2+s\left[\frac{5}{RC}\right]+\left[\frac{5+K}{R^2C^2}\right]}$$

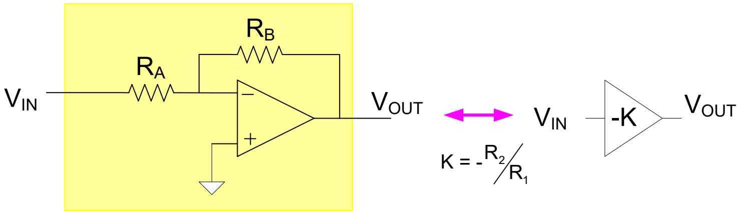
$$\omega_0 = \frac{\sqrt{5+K}}{RC} \qquad Q = \frac{\sqrt{5+K}}{5}$$

Example: 2nd Order -KRC Lowpass

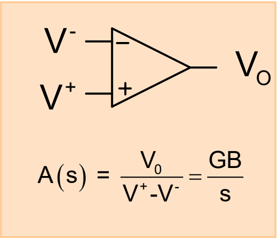




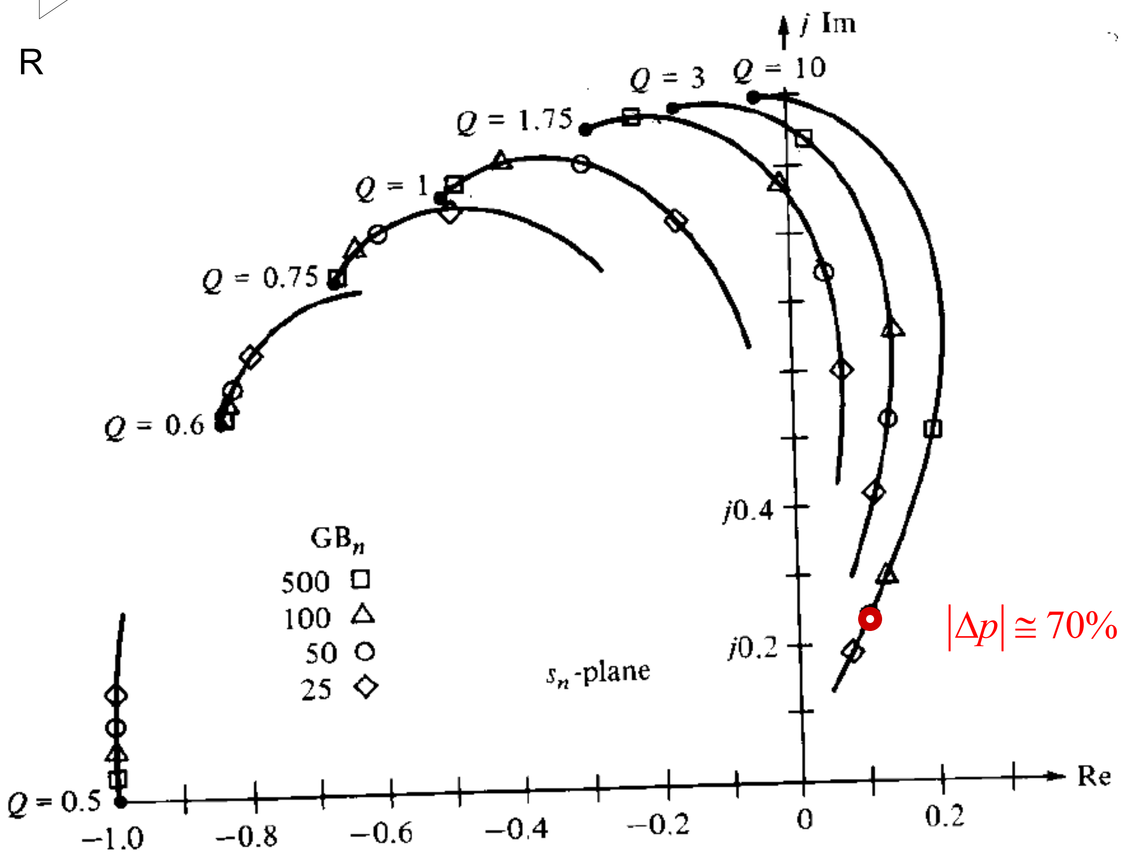
$$\omega_0 = \frac{\sqrt{5+K}}{RC} \qquad Q = \frac{\sqrt{5+K}}{5}$$



Assume either R_A=R or R_A very large



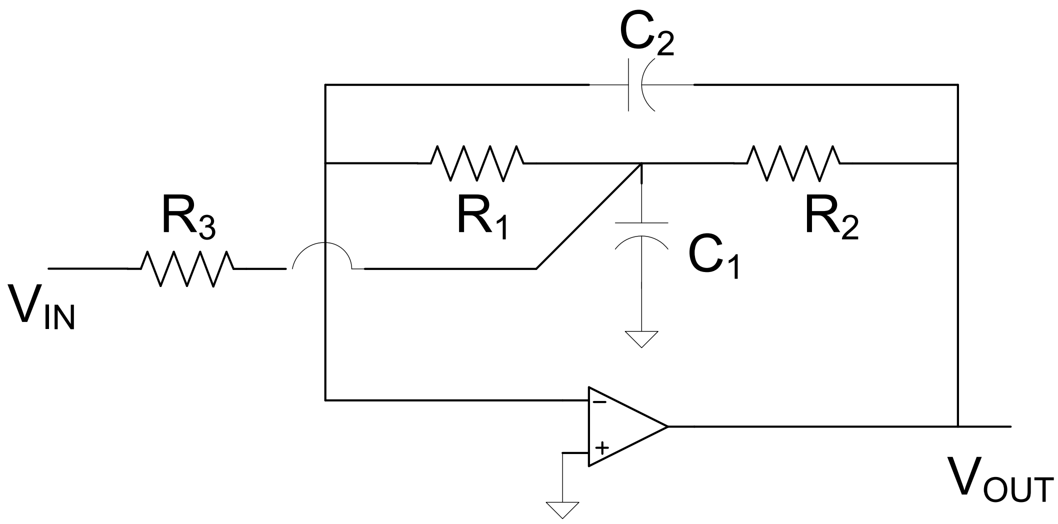
$$K(s) = -\frac{K_0}{1+\frac{(1+K_0)s}{GB}}$$



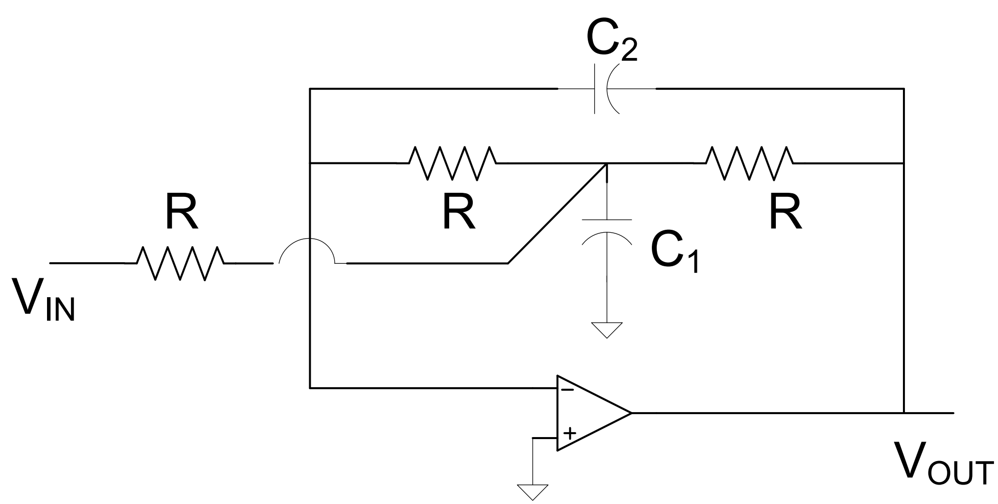
Poles "move" towards RHP as GB degrades

Even very large values of GB will cause instability

Example: 2nd Bridged-T FB Lowpass



$$T(s) = -\frac{\frac{1}{R_2R_3C_1C_2}}{s^2 + s\frac{1}{C_1}\left[\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right]+\frac{1}{R_1R_2C_1C_2}}$$

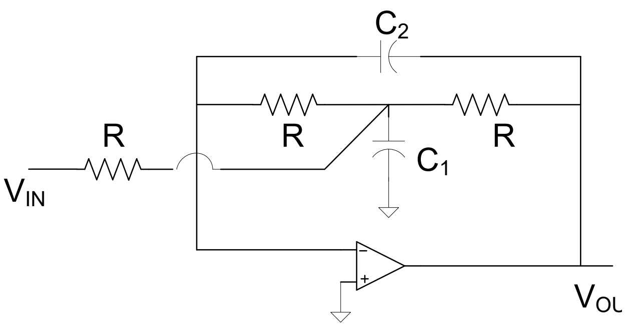


Equal R

$$T(s) = -\frac{\frac{1}{R^2C_1C_2}}{s^2 + s\left(\frac{3}{RC_1}\right)+\frac{1}{R^2C_1C_2}}$$

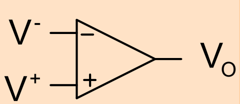
$$\omega_0 = \frac{1}{R\sqrt{C_1C_2}} \qquad Q = \frac{1}{2}\sqrt{\frac{C_1}{C_2}}$$

Example: 2nd Bridged-T FB Lowpass

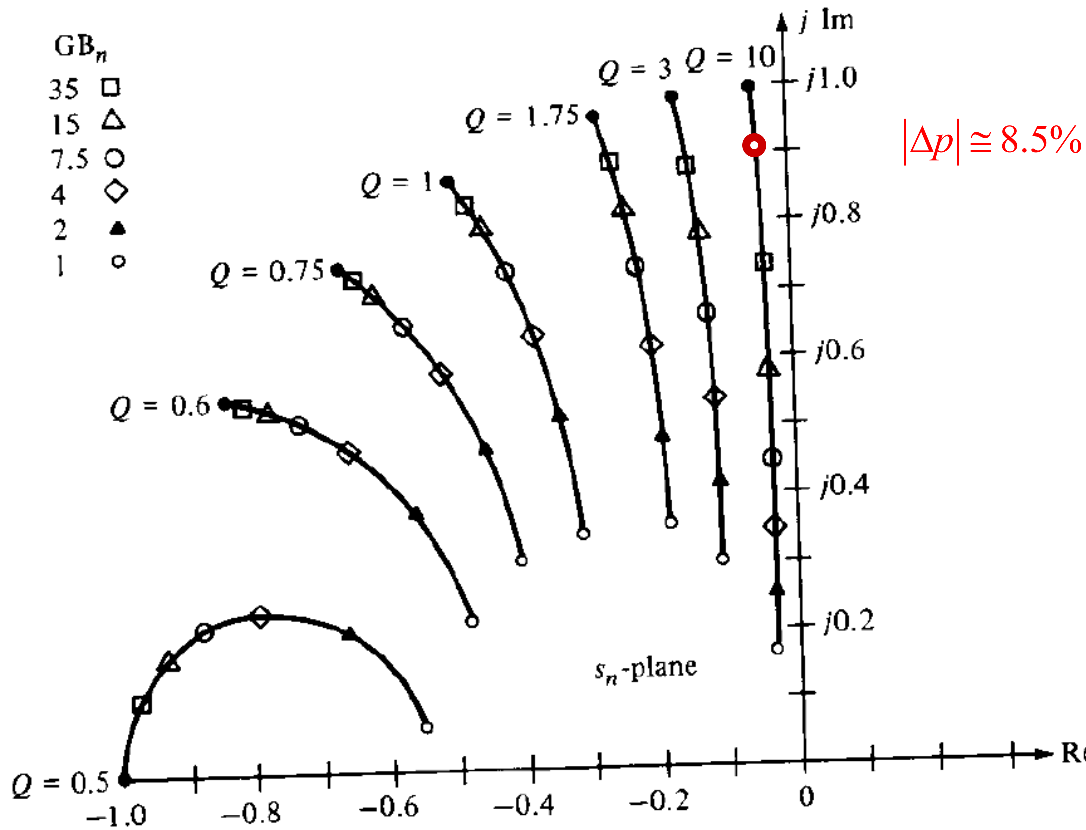


$$\omega_0 = \frac{1}{R\sqrt{C_1 C_2}} \qquad Q = \frac{1}{2}\sqrt{\frac{C_1}{C_2}}$$

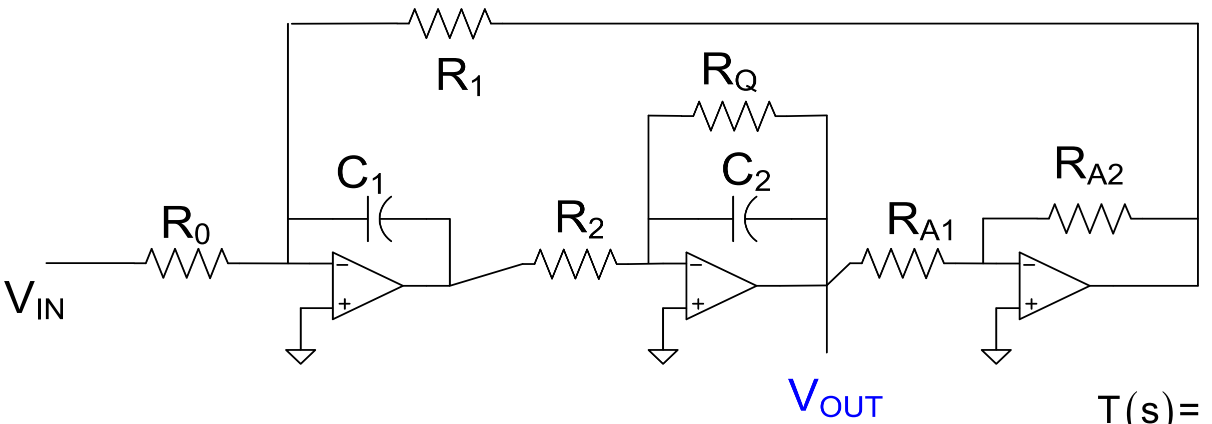
consider ⊙ ⟷ $GB_n = \frac{GB}{\omega_0} = 100$



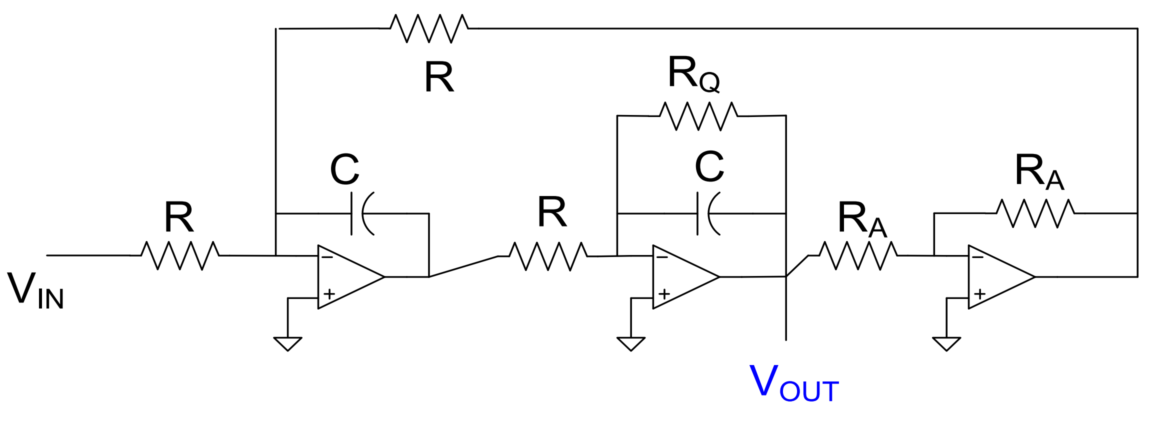
$$A(s) = \frac{V_0}{V^+ - V^-} = \frac{GB}{s}$$



Example: 2nd Two-Integrator-Loop Lowpass



$$T(s) = -\frac{\frac{1}{R_0 R_2 C_1 C_2}}{s^2 + s\left(\frac{1}{C_2 R_Q}\right) + \frac{R_{A2}/R_{A1}}{R_1 R_2 C_1 C_2}}$$

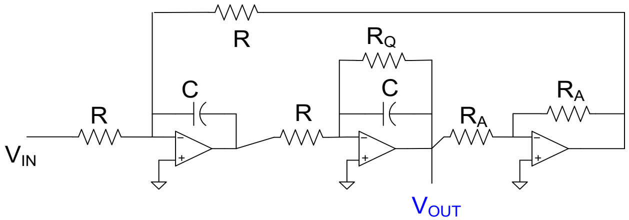


Equal R, Equal C (except R_Q)

$$T(s) = -\frac{\frac{1}{R^2 C^2}}{s^2 + s\left(\frac{1}{CR_Q}\right) + \frac{1}{R^2 C^2}}$$

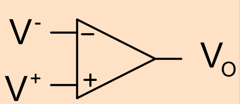
$$\omega_0 = \frac{1}{RC} \qquad Q = \frac{R_Q}{R}$$

Example: 2nd Two-Integrator-Loop Lowpass

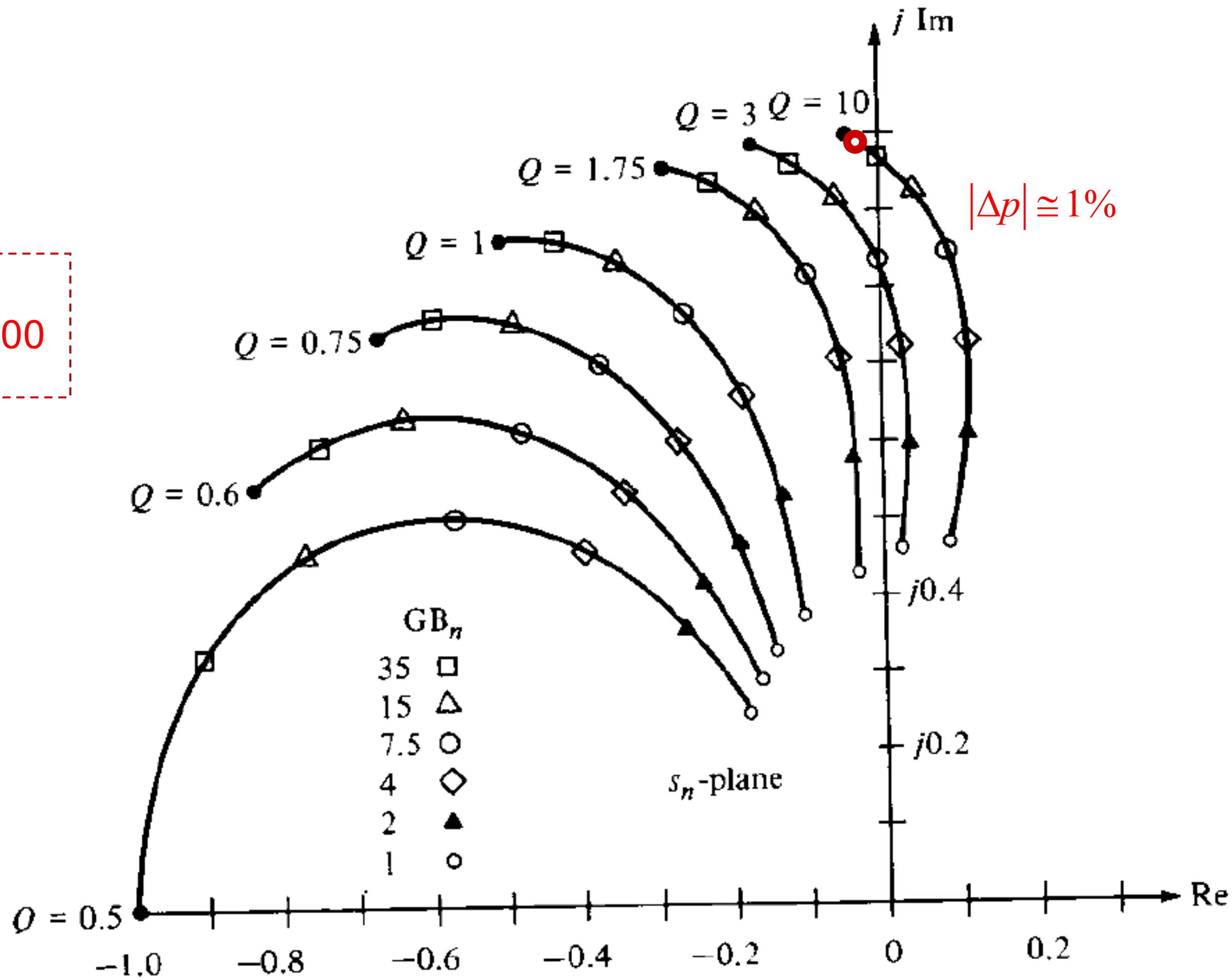


$$\omega_0 = \frac{1}{RC} \qquad Q = \frac{R_Q}{R}$$

consider ⊙ ⟷ $GB_n = \dfrac{GB}{\omega_0} = 100$

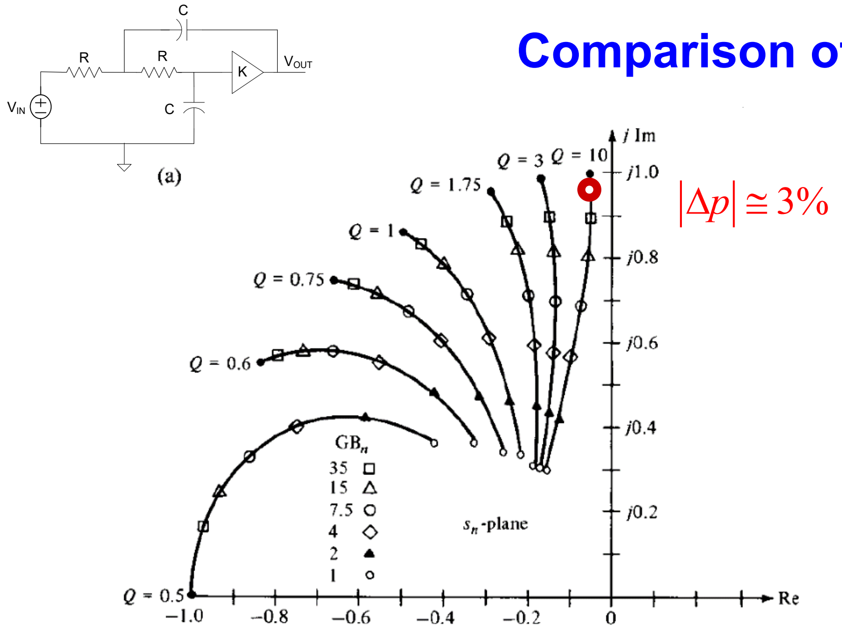


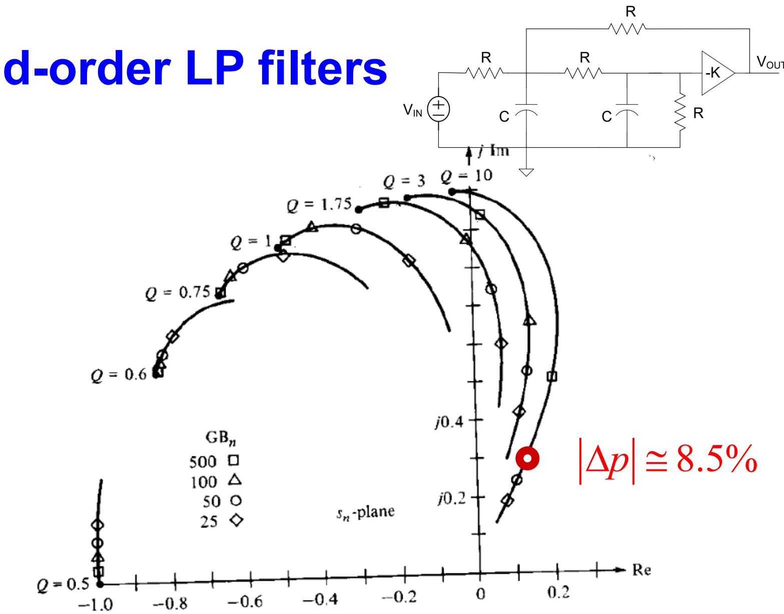
$$A(s) = \frac{V_0}{V^+ - V^-} = \frac{GB}{s}$$

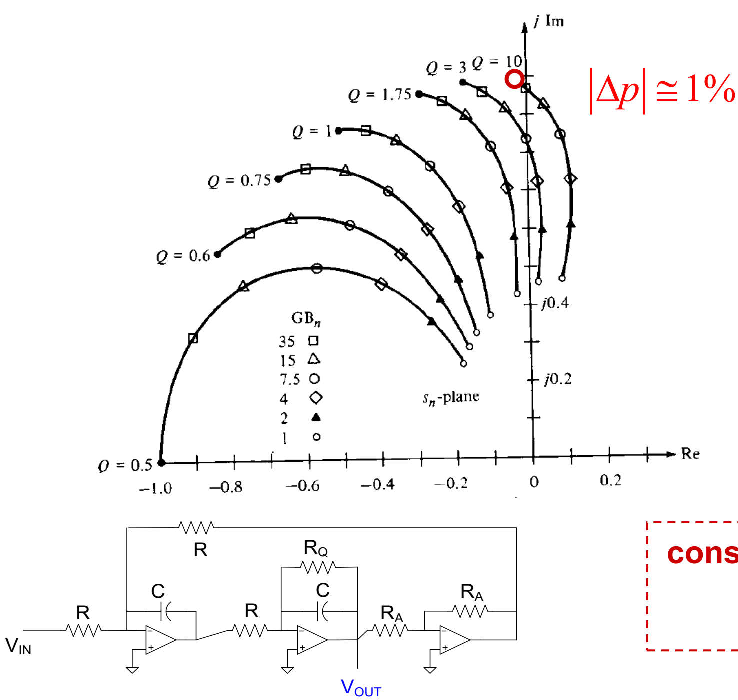


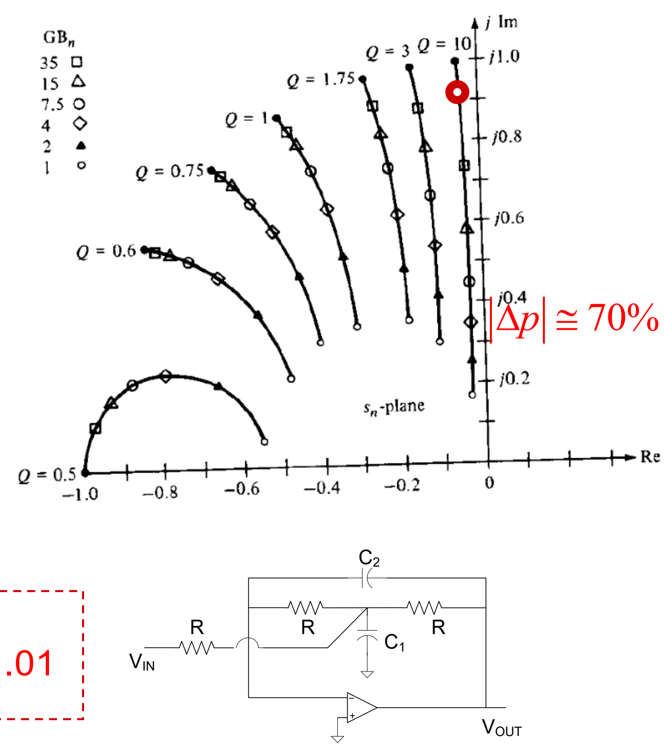
Poles "move" towards RHP as GB degrades

Comparison of 4 second-order LP filters









consider ● ⟷ $GB_n = \frac{GB}{\omega_0} = .01$

Some Observations

- Seemingly similar structures have dramatically different sensitivity to frequency response of the Op Amp

- Critical to have enough GB if filter is to perform as desired

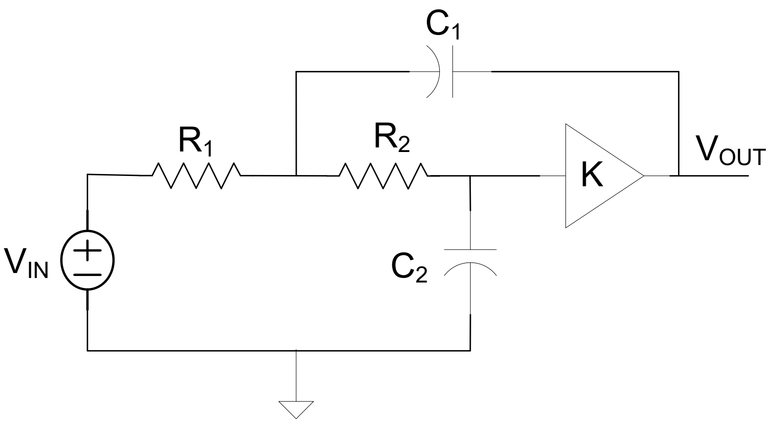
- Performance strongly affected by both magnitude and direction of pole movement

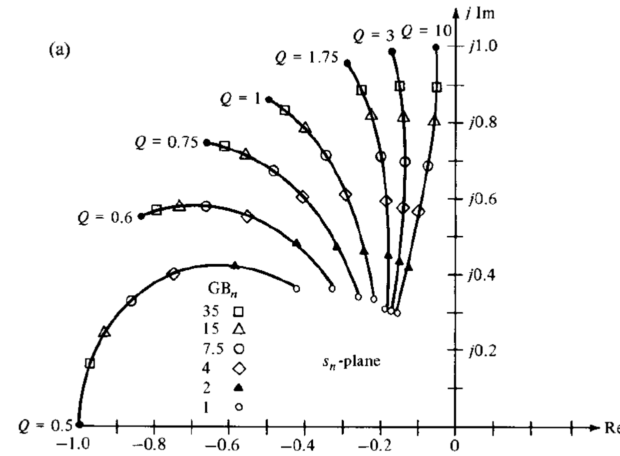
- Some structures appear to be totally impractical – at least for larger Q

- Different use of the Degrees of Freedom produces significantly different results

Sensitivity analysis is useful for analytical characterization of the performance of a filter

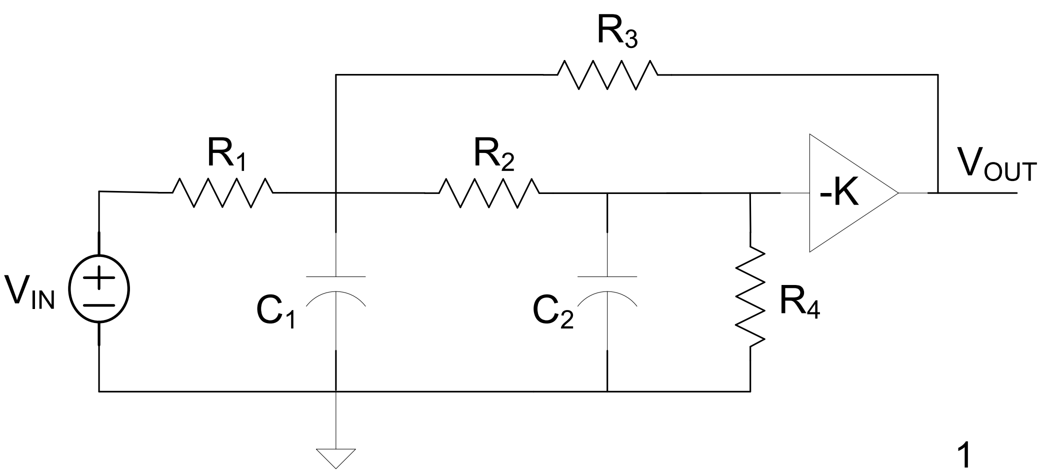
What causes the dramatic differences in performance between these two structures?
How can the performance of different structures be compared in general?

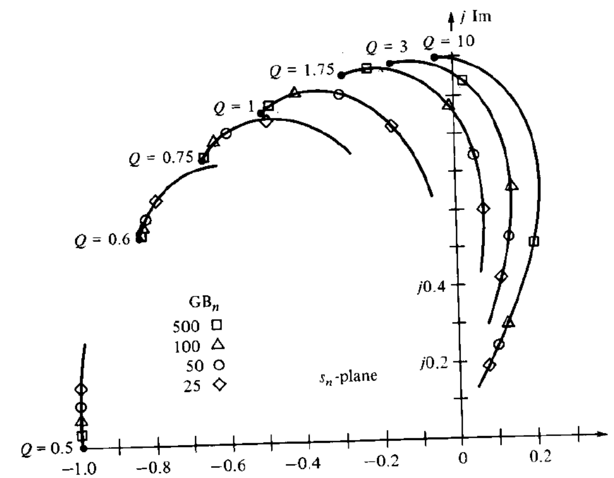




$$T(s)=K\frac{\frac{1}{R_1R_2C_1C_2}}{s^2+s\left[\frac{1}{R_1C_1}+\frac{1}{R_2C_1}+\frac{1-K}{R_2C_2}\right]+\frac{1}{R_1R_2C_1C_2}}$$

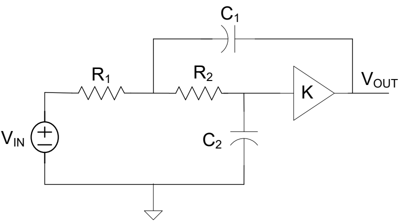
Equal R, Equal C, Q=10 Pole Locus vs GB_N

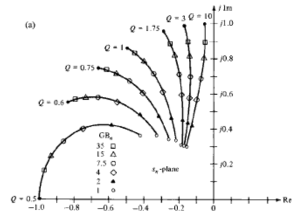




$$T(s)=-K\frac{\frac{1}{R_1R_2C_1C_2}}{s^2+s\left[\frac{1}{R_1C_1}\left(1+\frac{R_1}{R_3}\right)+\frac{1}{R_4C_2}+\frac{1}{R_2C_2}\left(1+\frac{C_2}{C_1}\right)\right]+\left[\frac{1+(R_1/R_3)(1+K)+(R_1/R_4)\left(1+(R_2/R_3)+(R_2/R_1)\right)}{R_1R_2C_1C_2}\right]}$$

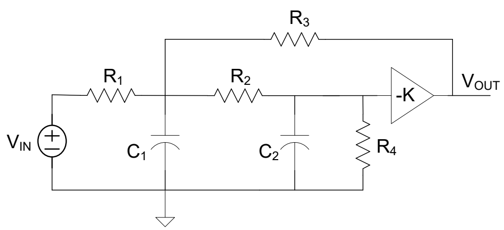
How can the performance of different structures be compared in general?

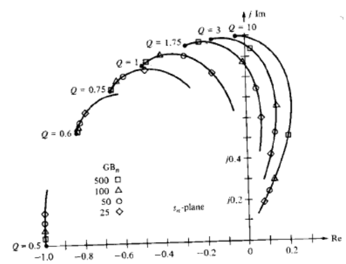




$$T(s)=K\frac{\frac{1}{R_1R_2C_1C_2}}{s^2+s\left[\frac{1}{R_1C_1}+\frac{1}{R_2C_1}+\frac{1-K}{R_2C_2}\right]+\frac{1}{R_1R_2C_1C_2}}$$

$$Q=\frac{1}{\sqrt{\frac{R_2C_2}{R_1C_1}}+\sqrt{\frac{R_1C_2}{R_2C_1}}+(1-K)\sqrt{\frac{R_1C_1}{R_2C_2}}}\qquad \omega_0=\frac{1}{\sqrt{R_1R_2C_1C_2}}$$





$$T(s)=-K\frac{\frac{1}{R_1R_2C_1C_2}}{s^2+s\left[\frac{1}{R_1C_1}\left(1+\frac{R_1}{R_3}\right)+\frac{1}{R_4C_2}+\frac{1}{R_2C_2}\left(1+\frac{C_2}{C_1}\right)\right]+\left[\frac{1+(R_1/R_3)(1+K)+(R_1/R_4)\left(1+(R_2/R_3)+(R_2/R_1)\right)}{R_1R_2C_1C_2}\right]}$$

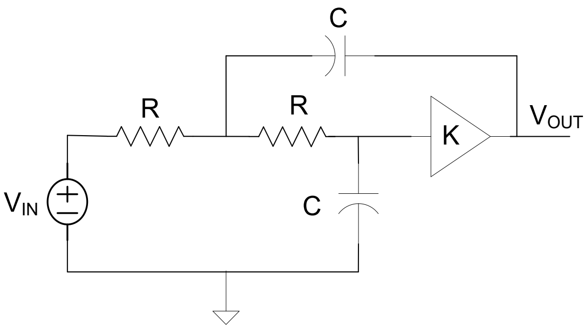
$$\omega_0=\sqrt{\frac{1+(R_1/R_3)(1+K)+(R_1/R_4)\left(1+(R_2/R_3)+(R_2/R_1)\right)}{R_1R_2C_1C_2}}$$

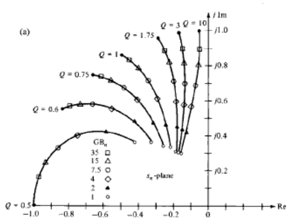
$$Q=\frac{\sqrt{\frac{1+(R_1/R_3)(1+K)+(R_1/R_4)\left(1+(R_2/R_3)+(R_2/R_1)\right)}{R_1R_2C_1C_2}}}{\frac{1}{R_1C_1}\left(1+\frac{R_1}{R_3}\right)+\frac{1}{R_4C_2}+\frac{1}{R_2C_2}\left(1+\frac{C_2}{C_1}\right)}$$

- **Equations for key performance parameters give little insight into the differences**
- **Expressions for key performance parameters quite complicated**

How can the performance of different structures be compared in general?

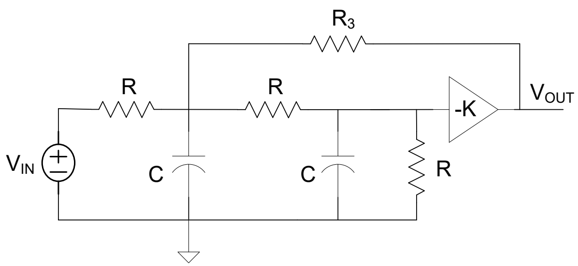
Equal R, Equal C implementations

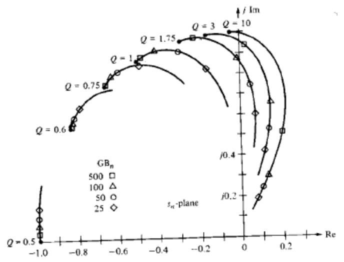




$$T(s)=K\frac{\frac{1}{R^2C^2}}{s^2+s\left[\frac{(3-K)}{RC}\right]+\frac{1}{R^2C^2}}$$

$$Q=\frac{1}{3-K} \qquad \omega_0=\frac{1}{RC}$$



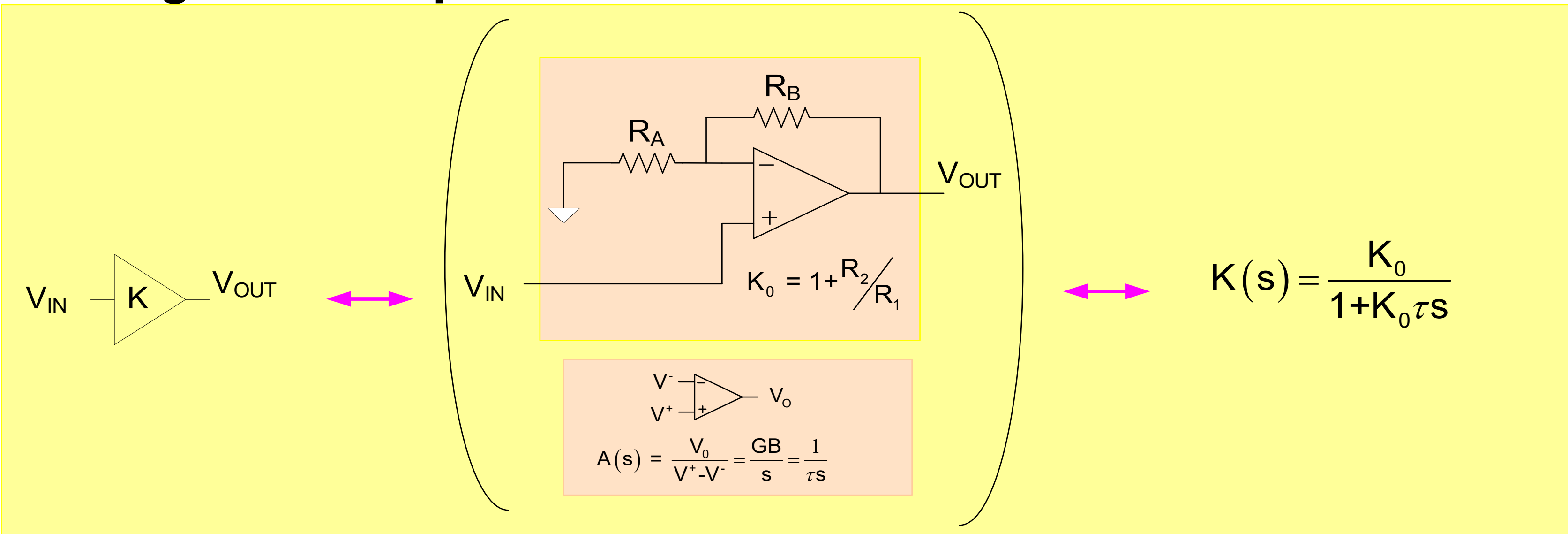


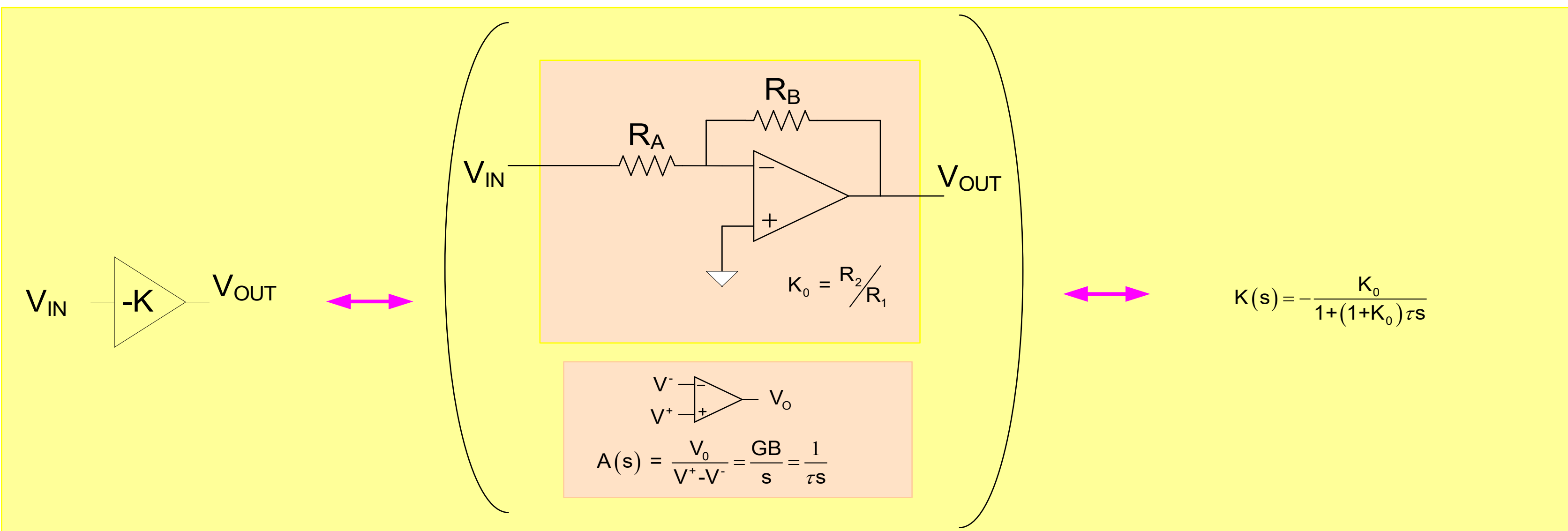
$$T(s)=-K\frac{\frac{1}{R^2C^2}}{s^2+s\left[\frac{5}{RC}\right]+\left[\frac{5+K}{R^2C^2}\right]}$$

$$Q=\frac{\sqrt{5+K}}{5} \qquad \omega_0=\frac{\sqrt{5+K}}{RC}$$

- **Analytical expressions for ω_0 and Q much simpler**
- **Equations for key performance parameters give little insight into the differences**
- **Effects of individual components is obscured in these expressions**
- **GB effects absent in this analytical formulation !!!!**

Modeling of the Amplifiers

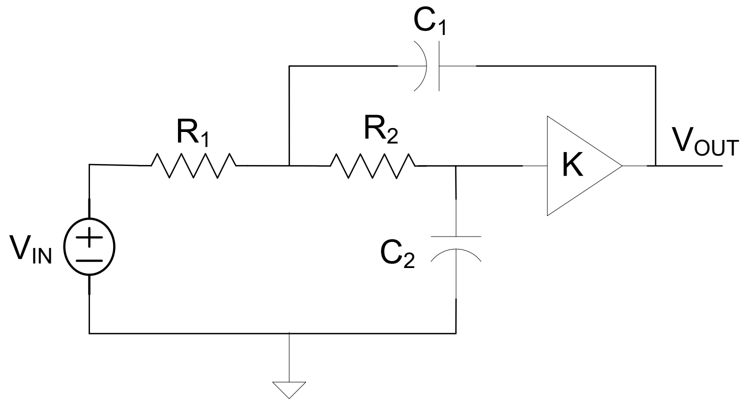




Different implementations of the amplifiers are possible

Have used the op amp time constant in these models $\tau = GB^{-1}$

GB effects in +KRC Lowpass Filter



$$T(s)=K\frac{\frac{1}{R_1R_2C_1C_2}}{s^2+s\left[\frac{1}{R_1C_1}+\frac{1}{R_2C_1}+\frac{1-K}{R_2C_2}\right]+\frac{1}{R_1R_2C_1C_2}}$$

$$K(s)=\frac{K_0}{1+K_0\tau s}$$

$$T(s)=\frac{\frac{K_0}{R_1R_2C_1C_2}}{s^2+s\left[\frac{1}{R_1C_1}+\frac{1}{R_2C_1}+\frac{(1-K_0)}{R_2C_2}\right]+\frac{1}{R_1R_2C_1C_2}+K_0\tau s\left(s^2+s\left[\frac{1}{R_1C_1}+\frac{1}{R_2C_1}+\frac{1}{R_2C_2}\right]+\frac{1}{R_1R_2C_1C_2}\right)}$$

$$\omega_0=\frac{1}{\sqrt{R_1R_2C_1C_2}} \qquad Q=\frac{1}{\sqrt{\frac{R_2C_2}{R_1C_1}}+\sqrt{\frac{R_1C_2}{R_2C_1}}+(1-K)\sqrt{\frac{R_1C_1}{R_2C_2}}}$$

ω_0 and Q in these expressions are for ideal op amp

$$T(s)=\frac{K_0\omega_0^2}{s^2+s\left[\frac{\omega_0}{Q}\right]+\omega_0^2+K_0\tau s\left(s^2+s\left[\frac{\omega_0}{Q}\left(1+K_0Q\sqrt{\frac{R_1C_1}{R_2C_2}}\right)\right]+\omega_0^2\right)}$$

$$T(s)=\frac{\frac{K_0}{R_1R_2C_1C_2}}{D_I(s)+K_0\tau s(D_{RC0}(s))}$$

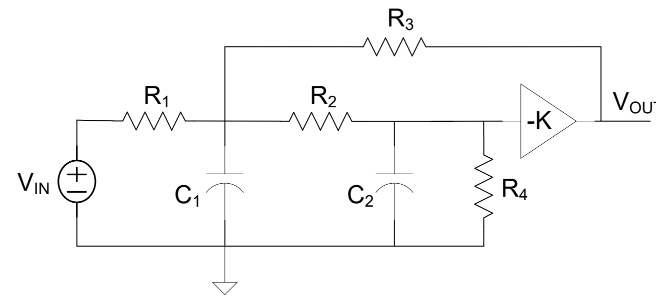
$D_I(s)$ is the D(s) if the OA is ideal

$D_{RC0}(s)$ is the D(s) of RC circuit with K=0

All linear performance effects can be obtained from this formulation

Op amp introduced an additional pole and moves the desired poles

GB effects in -KRC Lowpass Filter



$$T(s)=-K\frac{\dfrac{1}{R_1R_2C_1C_2}}{s^2+s\left[\dfrac{1}{R_1C_1}\left(1+\dfrac{R_1}{R_3}\right)+\dfrac{1}{R_4C_2}+\dfrac{1}{R_2C_2}\left(1+\dfrac{C_2}{C_1}\right)\right]+\left[\dfrac{1+(R_1/R_3)(1+K)+(R_1/R_4)\big(1+(R_2/R_3)+(R_2/R_1)\big)}{R_1R_2C_1C_2}\right]}$$

$$Q=\frac{\sqrt{\dfrac{1+(R_1/R_3)(1+K)+(R_1/R_4)\big(1+(R_2/R_3)+(R_2/R_1)\big)}{R_1R_2C_1C_2}}}{\dfrac{1}{R_1C_1}\left(1+\dfrac{R_1}{R_3}\right)+\dfrac{1}{R_4C_2}+\dfrac{1}{R_2C_2}\left(1+\dfrac{C_2}{C_1}\right)}$$

$$\omega_0=\sqrt{\frac{1+(R_1/R_3)(1+K)+(R_1/R_4)\big(1+(R_2/R_3)+(R_2/R_1)\big)}{R_1R_2C_1C_2}}$$

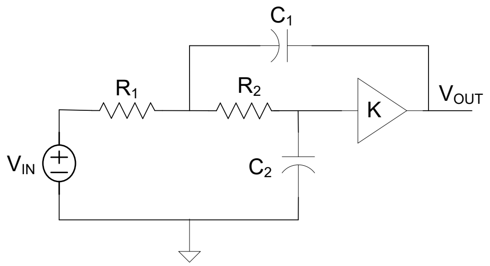
ω_0 and Q in these expressions are for ideal op amp

Now consider: $$K(s)=\frac{-K_0}{1+(1+K_0)\tau s}$$

$$T(s)=-K_0\frac{\dfrac{1}{R_1R_2C_1C_2}}{\left(s^2+s\left[\dfrac{1}{R_1C_1}\left(1+\dfrac{R_1}{R_3}\right)+\dfrac{1}{R_4C_2}+\dfrac{1}{R_2C_2}\left(1+\dfrac{C_2}{C_1}\right)\right]+\left[\dfrac{1+(R_1/R_3)(1+K_0)+(R_1/R_4)\big(1+(R_2/R_3)+(R_2/R_1)\big)}{R_1R_2C_1C_2}\right]\right)+\tau s(1+K_0)\left(s^2+s\left[\dfrac{1}{R_1C_1}\left(1+\dfrac{R_1}{R_3}\right)+\dfrac{1}{R_4C_2}+\dfrac{1}{R_2C_2}\left(1+\dfrac{C_2}{C_1}\right)\right]+\left[\dfrac{1+(R_1/R_3)+(R_1/R_4)\big(1+(R_2/R_3)+(R_2/R_1)\big)}{R_1R_2C_1C_2}\right]\right)}$$

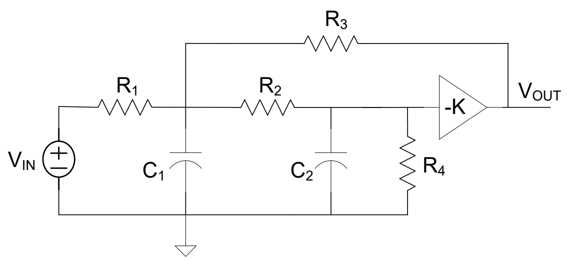
$$T(s)=\frac{\dfrac{-K_0}{R_1R_2C_1C_2}}{D_I(s)+(1+K_0)\tau s\big(D_{RC0}(s)\big)}$$

GB effects in KRC and -KRC Lowpass Filter



$$T(s)=\frac{K_0\omega_0^2}{s^2+s\left[\frac{\omega_0}{Q}\right]+\omega_0^2+K_0\tau s\left(s^2+s\left[\frac{\omega_0}{Q}\left(1+K_0Q\sqrt{\frac{R_1C_1}{R_2C_2}}\right)\right]+\omega_0^2\right)}$$

$$T(s)=\frac{\frac{K_0}{R_1R_2C_1C_2}}{D_I(s)+K_0\tau s\left(D_{RC0}(s)\right)}$$



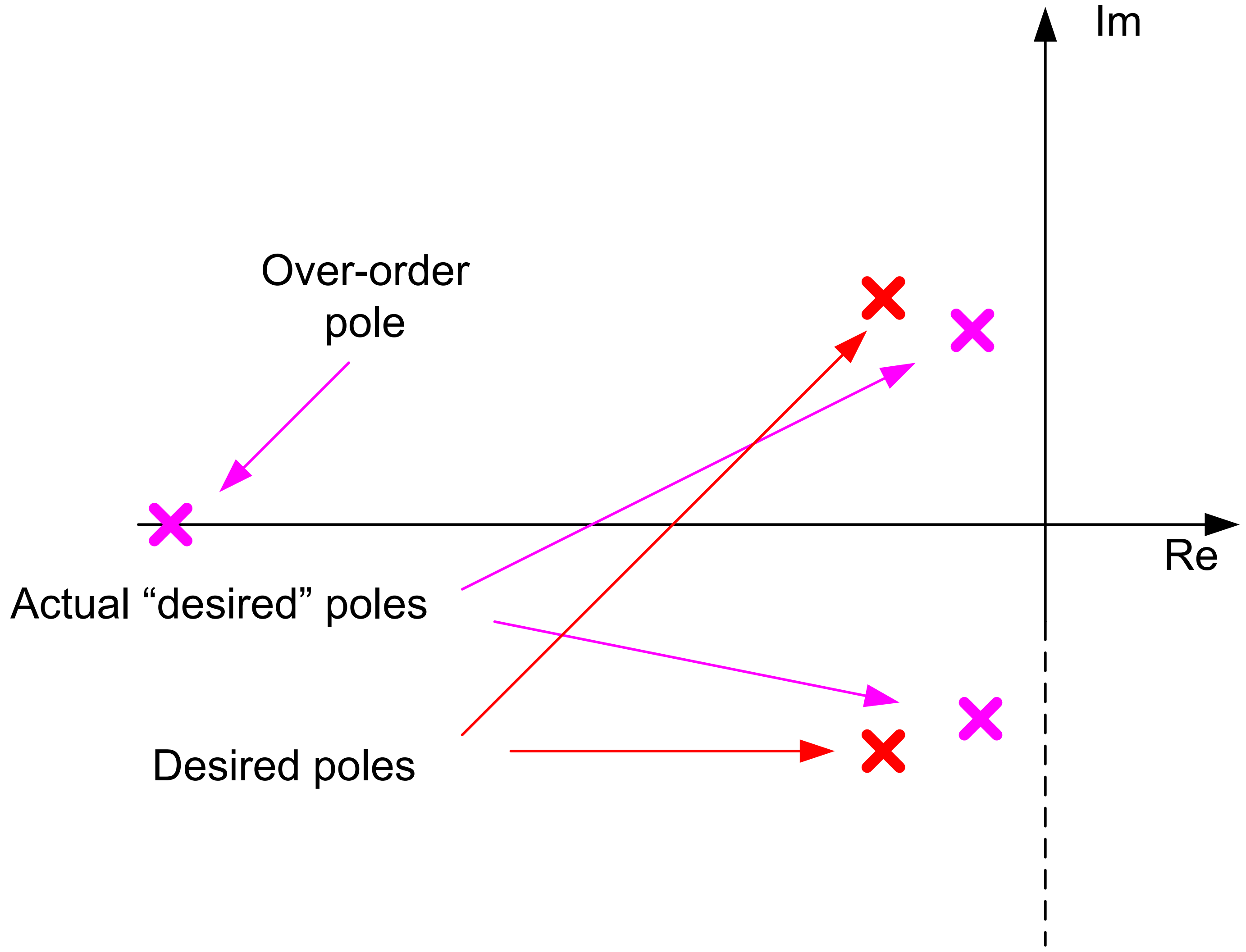
$$T(s)=-K_0\frac{\frac{1}{R_1R_2C_1C_2}}{\left(s^2+s\left[\frac{1}{R_1C_1}\left(1+\frac{R_1}{R_3}\right)+\frac{1}{R_4C_2}+\frac{1}{R_2C_2}\left(1+\frac{C_2}{C_1}\right)\right]+\left[\frac{1+(R_1/R_3)(1+K_0)+(R_1/R_4)\left(1+(R_2/R_3)+(R_2/R_1)\right)}{R_1R_2C_1C_2}\right]\right)+\tau s(1+K_0)\left(s^2+s\left[\frac{1}{R_1C_1}\left(1+\frac{R_1}{R_3}\right)+\frac{1}{R_4C_2}+\frac{1}{R_2C_2}\left(1+\frac{C_2}{C_1}\right)\right]+\left[\frac{1+(R_1/R_3)+(R_1/R_4)\left(1+(R_2/R_3)+(R_2/R_1)\right)}{R_1R_2C_1C_2}\right]\right)}$$

$$T(s)=\frac{\frac{-K_0}{R_1R_2C_1C_2}}{D_I(s)+(1+K_0)\tau s\left(D_{RC0}(s)\right)}$$

All linear performance effects can be obtained from this formulation

Op amp introduced an additional pole and moves the desired poles

Effects of GB on poles of KRC and -KRC Lowpass Filters



Stay Safe and Stay Healthy !

End of Lecture 18